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A Contribution to the Empirics of Total Factor Productivity's Theory Based on a CES Production Function-Case of Moroccan Economy

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SUMMARY: Some propositions related to about growth theory are not valid for all production functions. This is the case for the estimation of total factor productivity based on the growth accounting's approach which is easily verified with a Cobb-Douglas production function, but difficult to obtain with a CES production function. In this paper, we will try to do that estimation using Moroccan data.

KEYWORDS: Cobb-Douglas production function, CES production function, constant returns to scale, total factor productivity.

INTRODUCTION

A production function is a relationship representing a production technology that combines inputs, including capital and labor and any other factor that may affect the production process, to produce output(s) efficiently. This production function can take different forms, the best known is the Cobb-Douglas function. Other forms of production function exist in the economic literature, but they are less popular because they are difficult to handle mathematically, especially for estimating total factor productivity (TFP). Consequently, almost all researches on TFP use Cobb-Douglas production function, given the mathematical facilities that it allows, especially for implementing the growth accounting approach.

This paper completes a series of works that aim to estimate the TFP by applying the growth accounting approach on some production functions. Thus, the first paper was devoted to a Cobb-Douglas production function (OULAD EL FAKIR, 2022a), while the second paper was devoted to a translog production function (OULAD EL FAKIR, 2022b). In this work, we will try to extend our research to a production function with "Constant Elasticity of Substitution" (CES). This extension is of considerable interest to verify the applicability of the growth accounting approach to several production functions.

For that, we will try, in this paper, to make theoretical investigations on the TFP's estimation from a CES production function at the macroeconomic level with annual data from 1999 to 2019 for Moroccan economy. Also, we will highlight some handling difficulties that justify the omnipresence of the Cobb-Douglas production function in the growth accounting approach. Thus, this work is presented as follows: a first part will be reserved for a review of the literature on the different forms of a CES function. Then, some properties of the CES function are presented. A third part is reserved for the determination of the TFP for the case of a CES production function and, finally, a fourth part will be reserved for a numerical application on Moroccan data.

1. Review of the Literature

The CES function has been used extensively to analyze economic growth to the point where some authors have parameterized economic growth with values of the elasticity of substitution (CHILARESCU, 2019). Similarly, different formulations of the CES production function have been developed in the economic literature including those presented by (KLUMP & PREISSLER, 2000). Note that the variables and parameters used in the equations below have the same meaning throughout this article and are as follows:

- Y: output or income or value added.
- K: capital stock.
- L: employed labor force.
- B is an efficiency parameter whose change causes income's change in the same direction.

- λ is a distribution parameter representing the share of capital income in national income $\left(\lambda = \frac{Y-W}{Y}\right)$ where W wages and (1 - λ) represents the share of wages in national income. Thus, λ reflects the income's sharing between K and L (0 < λ <1);
- ρ is a substitution parameter that reflects the elasticity of substitution between variable $\left(\sigma = \frac{1}{1+\rho} \text{ with } \rho \geq -1\right)$;
- ν is a homogeneity parameter, also known as a yield parameter. Note that if ν > 1, then the return to scale is increasing, decreasing if 0 ≤ ν < 1 and if ν = 1 we have constant returns to scale (CRS).

a- Formulation of Pitchford (1960)[1]

$$Y = (a * K^{\rho} + b * L^{\rho})^{\frac{1}{\rho}} \qquad (1)$$

Where a and b are income distribution parameters.

b- Formulation of Arrow et al (1961)[2]
Arrow, Chenery, Minhas and Solow (ACMS) introduced what has become the standard specification of the CES production function which contains a substitution parameter, ρ, a distribution parameter, λ (0 < λ < 1) and an efficiency parameter, B :

$$Y = B * [\lambda * K^{\rho} + (1-\lambda) * L^{\rho}]^{\frac{1}{\rho}} \qquad (2)$$

c- Formulation of David and van de Klundert (1965)[3]

$$Y = [(B * K)^{\rho} + (A * L)^{\rho}]^{\frac{1}{\rho}} \qquad (3)$$

d- Formulation of Barro and Sala-i-Martin (1995)[4]

$$Y = B * \{\lambda * (A * K)^{\rho} + (1-\lambda) * [(1-A) * L]^{\rho}\}^{\frac{1}{\rho}} \qquad (4)$$

With:

$$\rho = \frac{\sigma - 1}{\sigma} \quad or \quad \sigma = \frac{1}{1+\rho} \qquad (5)$$

e- Production function with variable elasticity of substitution (VES) (REVANKAR, 1971) :
Liu and Hildebrand[5] formulated a variable elasticity production function as follows:

$$\mathrm{Y} = \mathrm{B} * \left[(1-\lambda) * K^{\rho} + \lambda * K^{m*\rho} * L^{(1-m)*\rho}\right]^{\frac{1}{\rho}} \qquad (6)$$

For this function, the elasticity of substitution is given by:

$$\sigma = \frac{1}{1 - \rho + \frac{m * \rho}{S_K}} \qquad (7)$$

where m is a parameter and S_K is the share of capital in income. According to relation (7), when m = 0, this function becomes a CES function.

f- Constant elasticity of substitution (CES) production function
This CES production function has the following general form (KMENTA, 1967):

$$\mathrm{Y} = B * [\lambda * K^{-\rho} + (1-\lambda) * L^{-\rho}]^{-\frac{\nu}{\rho}} \qquad (8)$$

These different formulations of the CES production function vary according to different restrictions imposed on the (distributional and/or efficiency) parameters.
Similarly, a review of the literature on techniques to estimate the parameters of a CES production function was presented in (THURSBY, 1980).

[1] Pitchford, J. D. (1960) : Growth and the elasticity of substitution. Economic Record ; 36 ; 491-503.
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2. Some Characteristics of the CES Production Function

The form of the CES function given in (8) will be analyzed in this work and it will be replaced, if necessary, by the form corresponding to constant returns to scale. So, the unknown parameters of this function are B, ρ, λ, ν and σ which must be estimated.

For a CES production function given in (8), we have :

$$F(t*K, t*L) = B*[\lambda*(t*K)^{-\rho} + (1-\lambda)*(t*L)^{-\rho}]^{-\frac{\nu}{\rho}} = t^{\nu}*F(K,L)$$

Thus, we can conclude that this production function is homogeneous of degree v.

When v = 1, we have constant returns to scale and the CES production function is written as follows:

$$Y = B*[\lambda*K^{-\rho} + (1-\lambda)*L^{-\rho}]^{-\frac{1}{\rho}} \qquad (9)$$

This function is continuous, increasing in K and L and with increasing marginal productivities since we have:

$$\begin{cases} F'_K = \frac{\delta Y}{\delta K} = \lambda*\nu*K^{-\rho-1}*B^{-\frac{\rho}{\nu}}*Y^{\frac{\rho}{\nu}+1} > 0 \\ F'_L = \frac{\delta Y}{\delta L} = (1-\lambda)*\nu*L^{-\rho-1}*B^{-\frac{\rho}{\nu}}*Y^{\frac{\rho}{\nu}+1} > 0 \end{cases}$$

Thus, the marginal productivities of capital (F'_K) and labor (F'_L) are positive.

3. Determining TFP from a CES Production Function

In this part, we will try to decompose economic growth rate according to the contribution of the inputs and the TFP by using a CES production function.

Let a production function with constant elasticity of substitution for the case of two factors, capital and labor. This function is given in (9) which is not linear in logarithms. Because of this difficulty, we will resort to a transformation used by (BHATTACHARYA, 2017). For this, we will pose: $y = \frac{Y}{L}$ and $k = \frac{K}{L}$ and we obtain the intensive form of the production function. Thus, equation (9) becomes:

$$y = \frac{Y}{L} = \frac{1}{L}*B*[\lambda*K^{-\rho} + (1-\lambda)*L^{-\rho}]^{-\frac{1}{\rho}} = B*[\lambda*k^{-\rho} + (1-\lambda)]^{-\frac{1}{\rho}} \qquad (10)$$

We take the log of both sides and get:

$$Lny = LnB - \frac{1}{\rho}*ln[\lambda*k^{-\rho} + (1-\lambda)]$$

We derive this equation with respect to time and get:

$$G = \frac{\dot{y}}{y} = \frac{\dot{B}}{B} + \frac{\lambda*k^{-\rho-1}*\dot{k}}{\lambda*k^{-\rho} + (1-\lambda)} \qquad (11)$$

with (.) denoting a differentiation with respect to time t.

Thus, from equation (11) we can see that:

$$G = \frac{\dot{B}}{B} + D \qquad (12)$$

and

$$\frac{\dot{B}}{B} = G - D \qquad (13)$$

with: $G = \frac{\dot{y}}{y}$ and $D = \frac{\lambda*k^{-\rho-1}*\dot{k}}{\lambda*k^{-\rho}+(1-\lambda)}$

G is called the growth rate of output per worker and it is equal to the sum of two components, namely, the Solow residual corresponding to the CES production function $\left(\frac{\dot{B}}{B}\right)$ and the capital deepening component corresponding to the CES production function (D).

4. Application to the Case of the Moroccan economy

Since the CES production function is not linear and is not subject to simple mathematical transformations, Kmenta used the Taylor expansion around the point ρ = 0 to ease the form of this function and excluded the terms that have a power in ρ greater than 2 (ARDENTI & REICHENBACH, 1972).

$$Ln(Y) = Ln(B) + \nu\lambda * LnK + \nu * (1-\lambda) * LnL - \frac{1}{2} * \rho * \nu * (1-\lambda) * [LnK - LnL]^2 \quad (14)$$

This form, known as the "Kmenta approximation of the CES production function" (MISHRA, 2006), will be estimated by the ordinary least squares (OLS) method and we obtain:

$$Ln(Y) = 4{,}72 + 0{,}46 * Ln(K) + 0{,}11 * Ln(L) + 0{,}05 * [Ln(K) - Ln(L)]^2 \qquad R = 0{,}999 \quad et \quad DW = 1{,}2 \quad (15)$$

This gives:

$$\begin{cases} B = 112{,}2 \\ \lambda = 0{,}8 \\ \nu = 0{,}57 \\ \rho = -1{,}1 \end{cases}$$

With:

- Y: value added (in Millions DH of 2007);
- K: capital stock (in Million DH of 2007);
- L: employed labor force (in persons);

Here, we found that $\rho = -1{,}1$ or $\rho = -1$ and this value matches perfectly with the ρ's lower boundary. Also, it should be noted that the value found for ν (ν = 0.57) indicates that this CES production function is with decreasing returns to scale. This is totally opposite to the result obtained for the case of a Cobb-Douglas production function where we found a production function with increasing returns to scale for the Moroccan economy (OULAD EL FAKIR, 2022a).

After estimating the different parameters, equation (8) becomes as follows:

$$Y = 112{,}2 * [0{,}8 * K^{1{,}1} + 0{,}2 * L^{1{,}1}]^{0{,}52} \quad (16)$$

We will now move on to a second situation where we will estimate equation (9), that is, assuming a CES production function with constant returns to scale (ν = 1). For this, we will return to the approximation of Kmenta given by equation (14) which becomes (after replacing ν by 1):

$$Ln(Y) = Ln(B) + \lambda * LnK + (1-\lambda) * LnL - \frac{1}{2} * \rho * (1-\lambda) * [LnK - LnL]^2 \quad (17)$$

Equation (17) can be written as follows:

$$Ln(Y) - LnL = Ln(B) + \lambda * (LnK - LnL) - \frac{1}{2} * \rho * (1-\lambda) * [LnK - LnL]^2$$

Adopting the intensive form, this equation becomes:

$$Lny = LnB + \lambda * Lnk - \frac{1}{2} * \rho * (1-\lambda) * [Lnk]^2 \quad (18)$$

Solving this equation by ordinary least squares gives:

$$Lny = -2{,}22 + 0{,}43 * Lnk + 0{,}05 * [Lnk]^2 \quad (19)$$
$$R = 0{,}99 \quad et \quad DW = 1{,}1$$

This gives us:

$$\begin{cases} B = 0{,}1 \\ \lambda = 0{,}4 \\ \rho = -0{,}2 \end{cases}$$

We then get:

$$Y = 0{,}1 * [0{,}4 * K^{0{,}17} + 0{,}6 * L^{0{,}17}]^{5{,}9} \quad (20)$$

To calculate $\frac{\dot{B}}{B}$, we will return to the calculation of B, for each year, by:

$$B_t = \frac{Y_t}{\left[0{,}4 * K_t^{0{,}17} + 0{,}6 * L_t^{0{,}17}\right]^{5{,}9}}$$

Finally, we find:

$G = 3{,}1\%$ and $\frac{\dot{B}}{B} = -2{,}2\%$

Returning to relationship (12), we can say that for an average growth rate of about 3.1%, TFP contributes negatively with -2.2%. The gap has been filled by capital deepening which allowed a very high output per worker reaching 5.3% for the period 1999-2019.

Thus, the growth of TFP obtained through a CES production function (-2.2%) is paradoxically opposed to that obtained through the use of a Cobb-Douglas production function, which was around 1.1%. (OULAD EL FAKIR, 2022a).

To sum up, we can say that, through the results obtained by this series of researches on the applicability of the growth accounting approach to different production functions to estimate TFP, if this task is easy with a Cobb-Douglas production function, it is hard with a CES function and it is harder with a translog production function

As for the effect of TFP on growth, we have seen that, based on Moroccan data, it can be positive (with a Cobb-Douglas production function), negative (with a CES production function) and inconclusive (with a translog production function).

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DATA SOURCES

All data are taken from the Haut Commissariat du Plan's web site (www.hcp.ma).

[6] N.B : Papers cited in references were consulted while those appearing in footers are cited by the authors in references and appear in this paper only to give readers more reading materials.